The Applicability of Non-Cooperative Game Theory in Transport Systems Analysis

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Abstract

Various models that incorporate concepts from Non-Cooperative Game Theory are described in the transport literature. Game Theory provides powerful tools for analysing transport systems, but these tools have some drawbacks that should be recognized. In the current paper we review games that describe transport problems and discuss them within a uniform context. Although the paper does not introduce new tools, it presents insights concerning the relations between transport models and games. We divide existing games into groups and show that some common features characterise multiple games. We distinguish between games that make a conceptual contribution and games that are suitable for application. Compact or symmetric game structures make remarkable observations but often do not support actual decision-making. Less aesthetic formats, most of which are Stackelberg games between authorities and travellers, are stronger as instruments that assist in determining real-life policies; these formulations can be treated by practitioners as mathematical programs with equilibrium constraints and not as games.
INTRODUCTION

Non-Cooperative Game Theory (NCGT) is an important source of models for policy making and behavioural studies. Among the applications of NCGT in a wide range of fields, various models have been proposed in the transport literature. NCGT provides powerful tools for analysing transport systems, because it deals with situations involving multiple decision makers, whose objectives fully or partially clash with each other; many transport problems fit into this area.

In the current paper we bring a comparative analysis of games that describe transport problems. Such analysis, which has not appeared in literature so far, is useful in identifying opportunities and risks involved in developing NCGT formulations to replicate transport phenomena. We are especially interested in investigating whether transport games are suitable for practical application as tools for decision-support.

Situations modelled as games typically involve several parties having different interests, who need to decide how to behave. The level of benefit that each party gains depends not only on its own actions, but also on the choices of the other parties. The mathematical formulation of all games is similar, either explicitly or implicitly, to an optimisation problem that includes more than one objective, and the decision variables are shared by the different objectives. Defining a game requires identification of the players, their alternative strategies and their objectives. Formulating a problem as a game is worthwhile if the solution, such as Nash equilibrium or Stackelberg equilibrium, leads to new insights on the analysed problem.

In this paper we classify games into groups in two ways. One way of categorisation distinguishes between concept games and instrument games. Concept games focus on small-scale cases of greater problems, and are meant to establish theoretical principles. Their key contribution is in the qualitative introduction of new ideas, and if they include a computational element, it is often of a discrete nature. Instrument games deal with full-size, realistic scenarios and are application-oriented; they are mainly aimed at determining values of quantitative, continuous variables.

The other method of grouping used here has to do with who the players are. We define four groups in this respect: games against a demon; games between travellers; games between authorities; and games between travellers and authorities. Each section of the paper examines one of the groups. Characteristic of all the reviewed games are summarised in table 2, and the comparison between the groups is summarised in table 3.

Note that it is not our intention to review the theory. For a discussion of the basics of NCGT, see Fudenberg and Tirole (1), Hilier and Lieberman (2) or Kreps (3). We do not bring particular details of the reviewed games, since our main interest is in concepts.

GAMES AGAINST A DEMON

In this section we group together problems formulated as zero-sum games, which are the most basic structure in NCGT. Games of this simple form assume a strict competition between players: there is only one objective function, that one player wishes to maximise and the other wants to minimise. We will demonstrate that this simple structure leads to a rather abstract definition of the players, which is paradoxically not particularly simple. Colony (4) formulates a route choice problem as a zero-sum game. One of the players is a driver that chooses whether to use an arterial road, where the traffic volume does not affect driving conditions, or a motorway, where heavier traffic results in a more disturbed drive. The other player is an imaginary entity, which chooses the level of service on the road, and tries to disturb the driver’s journey as much as possible. The driver does not know what level
of service is expected, and various assumptions regarding the state of uncertainty are made. Despite the choice simultaneity, not Nash equilibrium but other solution terms are used, and the chance for a reasonable level of service that makes the driver chance his/her choice is derived.

Bell (5) describes a zero-sum game between a driver, which chooses a driving path through a road network, and an evil entity, which chooses the costs of using the network links. The driver aims at minimising the journey cost, while the evil entity desires to increase it:

$$\max_i \left( \min_j \left( \sum_p p_i c_{ij} q_j \right) \right)$$

where \( i \) is an index for road links; \( j \) is an index for every feasible set of link costs; \( p_i \) is the chance that a driver chooses a path that goes through link \( i \); \( q_j \) is the chance that the evil entity chooses the set \( j \); and \( c_{ij} \) is the cost of driving through link \( i \) when the chosen set of costs is \( j \). Since the driver cannot anticipate the decision of the evil entity, the equilibrium cost is the cost he/she will be willing to compromise on under the pessimistic assumption of maximum obstructions. In an unreliable system the driver will compromise on a higher cost, hence the Nash equilibrium of the game is suggested as a measure for network reliability.

Bell and Cassir (6) extend this methodology to a case that involves multiple drivers, and includes user equilibrium across the road network. The evil entity is replaced with multiple demons, which annoy each origin-destination pair separately. The formulation contains a series of pairs of programming problems, solved for each origin-destination.

A similar concept is employed again by Bell (7) for a freight vehicle routing problem. A two-player, zero-sum game is defined between a dispatcher, who seeks the lowest cost vehicle route, and a demon, which has the power to cause a road link to fail. The solution is interpreted as the risk-averse expectation of a worst-case scenario. It is therefore observed that in all zero-sum transport games, travellers play against some sort of demon. Introducing an imaginary player is not uncommon in NCGT and can form a powerful addition if the tendency towards some behaviour is described as the will of this player. It should be noted, though, that a natural tendency to thwart a driver’s journey is a very strict assumption. Bell’s games (5-7) explain the demon’s motivation plausibly by noting that the solution gives a pessimistic perspective, which can be used as a reliability indicator. Colony’s game (4) lacks such explanation. All games against a demon are concept games, although Bell’s games can be used as instrument games in specific cases where there is interest in the consequences of worst-case scenarios. Note also that in all games of the current group, the decision maker is an external observer which is not one of the players; it is indirectly involved as it can alter features of the input network.

### GAMES BETWEEN TRAVELLERS

In this section we review games where all players are users of the transport system. The definition of the players here, unlike their definition in the previous group, is very straightforward.

Fisk (8) mentions that the user equilibrium principle, introduced by Wardrop (9), is in fact a game since it meets the conditions of Nash equilibrium: no driver can reduce his/her travel time by changing the path choice.

Rosenthal (10) formulates a general game between individuals who choose elements out of a given set, where the cost of each element increases if more individuals choose it. Rosenthal illustrates a specific case of this game, identical to a discrete relaxation of an assignment problem (although this is not explicitly stated). A programming problem is formulated, whose solution is always a pure-strategies Nash equilibrium of the game; it is shown that a solution
always exists. Rosenthal’s model is not frequently used for transport application, but it is often cited as a historical step towards recent NCGT developments.

Van Vugt et al (11) present a two-player strategic-form game, where each player chooses either car or public transport. The choice mechanism is illustrated in table 1 (where there is no importance to the exact values, only to the relations between them):

<table>
<thead>
<tr>
<th>Player 1</th>
<th>Public transport</th>
<th>Car</th>
</tr>
</thead>
<tbody>
<tr>
<td>Public transport</td>
<td>(4, 4)</td>
<td>(-4, 8)</td>
</tr>
<tr>
<td>Car</td>
<td>(8, -4)</td>
<td>(0, 0)</td>
</tr>
</tbody>
</table>

Table 1: A game by Van Vugt et al (11)

The left/right numbers in each cell are the utility value for player 1 / 2, respectively. The table shows that using car is the best option for any of the players only if the other player uses public transport, i.e. when there is no congestion. If both choose public transport, their utility values are high, because increased ridership enables service improvements. Still, the only Nash equilibrium is when both travellers choose the car; the conclusion is that the selfish way travellers make their choices is bad for everyone.

James (12) formulates a strategic-form game with N players. Each player is a traveller who decides whether or not to use a given road segment. The utility to any player from using the road decreases if there are many users. This is demonstrated in a two-player example that has two pure-strategies equilibrium points, both occur when one of the players decides to travel and the other one decides not to. The game does not predict which player will give up, and does not explain how the players in a symmetric game can reach asymmetric equilibrium. The game also has a more acceptable, symmetric mixed-strategies equilibrium, where the players have equal chances of choosing to travel. The inability to explain situations with several equilibrium points is a well-known deficiency of NCGT.

Levinson (13) presents games with two or three drivers who choose their departure times. The games explore the foundations of traffic congestion: when more than one driver chooses to depart at the same period, congestion occurs and one of the drivers arrives at the destination later than desired. Changes in the pure-strategy equilibrium, following the introduction of congestion pricing, are analysed under different input values of the players’ valuations of earliness, lateness and delay.

Pedersen (14) investigates the hypothesis that improved safety increases the number of drivers who behave aggressively. A game between two drivers is introduced, where each of them chooses their level of care when driving. Two types of drivers are defined: doves and hawks; whether the players are doves or hawks is required as input. In a situation where two doves meet, the game is solved as a Cournot game with Nash equilibrium, assuming that the drivers act simultaneously. An encounter between a hawk and a dove is solved as a Stackelberg game, where the hawk is the leader. When two hawks meet, the author suggests that they both try wrongly to behave as leaders, resulting in a state of no-equilibrium. Pedersen’s game employs NCGT conventions to model road safety in an original way, although it should be noted that some of the conclusions from the analysis stem directly from its assumptions.

Tay (15) demonstrates another safety issue through a two-player, strategic-form game. Both players are individuals intending to buy a car, and can choose a big or a small car; they wish to minimise the relative risk of driver fatal injury in a collision between cars. Data of such risk are presented, and it is shown that in an incident involving two small cars, the overall risk is the smallest; but when choosing a car, the players can reduce their own risk by buying
a big car. Equilibrium is only reached when both players buy big cars, which is an inferior outcome to the scenario where both choose small cars.

All games between travellers are concept games that illustrate various aspects of user equilibrium. Most of them (9-13) present the concept of a competition between travellers on a limited road space: each player prefers to have much space for their own use, and the entrance of a new user results in reduced utility for everyone. To deliver this idea freely from the computational difficulties that NCGT imposes, most games between travellers concentrate on a small network with a small number of players, whose behaviour is perfectly symmetric, as they all have equal objectives and equal strategies. Wardrop’s model (9), although its introduction as a game was proposed by Fisk (8) after years of implementation, also constitutes a strong instrument game. If information about the distribution of the level of aggressiveness across drivers is available, Pedersen’s game (14) too can be used as an instrument game. The games formulated by Van Vugt et al (11), James (12), Levinson (13) and Tay (15) discuss cases of the famous Prisoner’s Dilemma: all players would be better off if they did not act selfishly.

Similarly to games against a demon, the role that games between travellers can take in decision support is indirect, as none of the players is a transport policy maker. Even when the policy maker is virtually present in the game, as in Levinson’s congestion charge scenario (13), the policy decision is not presented as a set of strategies, and there is no specification of the policy objectives; no policy is therefore an explicit output of the game. The decision maker in all these games can examine what equilibrium results from different input parameters, for instance in an appraisal framework, but the games do not aim at advising which one of them is optimal.

GAMES BETWEEN AUTHORITIES

In the two remaining sections we discuss games where at least one of the players is an authority or a company. For our current needs it is not necessary to distinguish between official authorities (governmental, municipal, regional) and companies (such as operators of public transport services or toll roads), because the objective of both companies and authorities is based on summation of some value across the entire transport system. Note that games that involve both authorities and travellers have some special features and are therefore discussed separately in the next section. The group of games discussed here includes only two games, in which authorities play against each other without involving travellers.

Castelli et al (16) formulate a game between two authorities that have different responsibilities in a freight transport network. The first authority determines the flows on the network roads, aiming at minimising the overall transport cost. The other authority determines the capacities of the network roads, wishing to maximise its profit, which is proportional to the flow of freight traffic through the network. Two Stackelberg games are defined: in each of them, one of the players is the leader and the other is the follower. The examples of implementation brought by the authors are not entirely clear, and involve, for instance, treating the war in Yugoslavia in the 90’s as the profit-maximising player that determines the capacities.

Martín and Román (17) describe a game between several airlines that need to decide where to locate their hub and which direct services to operate. The game is played in two stages, which are in fact two separate games. First, the airlines sequentially choose their hub locations; when each airline enters the game, it can choose any location that has not been chosen yet by other airlines. In the second stage all airlines need to determine which direct services will be
given high frequencies; each airline aims at maximising its share in the market of flights between each pair of cities. The choice at this stage is made simultaneously by all airlines, and the solution is a Nash equilibrium. All players have the same objective, but since the choice of hubs in the first stage puts each airline at a different starting point, the game is not entirely symmetric. Although the game has a somewhat irregular structure, it seems a good example of a realistic model for commercial competition.

Since only two games between authorities were found in literature, it is not possible to reach general conclusions concerning this group. The small number of such games is surprising, considering that NCGT seems a natural tool for analysing relations between authorities; these relations can be either symmetrical, when several companies have similar objectives and strategies, or hierarchical, involving governmental authorities, local/regional authorities, private companies and/or entrepreneurs. A practical analysis of a semi-symmetrical competition game was presented above (17), and it seems that similar analysis could prove applicable to many cases where public transport operators compete on providing services. Trends such as tendering and privatisation, that have a vital role on today’s transport agenda, also seem apt to be modelled through games between authorities. Games between authorities can be either concept games or instrument games; the involvement of the decision maker can be either explicit or implicit.

GAMES BETWEEN TRAVELLERS AND AUTHORITIES

We illustrated that in games against a demon, the objective functions of the players are strictly the opposite of each other, and in games between travellers, the objectives of all players are equal. In the current section we discuss games between travellers and authorities (governmental or other); in such games, the objectives of the involved parties are different, but not rigorously contradictory.

Bjørnskau and Elvik (18) describe a strategic-form game between a driver and an authority that enforces traffic rules. The driver chooses whether to violate the legal speed limit and the authority chooses whether to deploy enforcing units. The authors present assumptions regarding the utility values, e.g.: drivers tend to violate the speed limit only when there is no enforcement. They calculate the probability of speed limit violation at the mixed-strategies equilibrium; their main claim relates to the fact that this probability does not depend on the penalty on breaking the speed limit. The authors conclude that the level of enforcement, rather than the penalties, should be increased to improve obedience. Note that although the equilibrium probability of each player indeed does not depend on this player’s levels of utility, we sometimes find this a serious flaw of NCGT, which cannot be used to prove the existence of any behaviour without robust empirical evidence.

Albert (19) presents a game between the operator of a toll road and a traveller. The operator decides on the level of toll it charges, and the traveller determines the maximum toll he/she is willing to pay. If the charged toll is less than this maximum, the traveller uses the road, otherwise – he/she uses public transport. If the traveller uses the toll road, his/her level of utility depends on the total number of road users; a required input to the game is a demand curve, which determines the number of road users as a function of the toll. The road operator wishes to maximise its revenue, which depends on the toll and on the number of users. The operator and the traveller make simultaneous choices, and the solution is a pure-strategies Nash equilibrium. Governmental authorities do not explicitly participate in Albert’s game, but their ability to implicitly influence the players’ choices by changing the public transport level of service is examined.
The traveller in Albert’s game is defined as a marginal traveller, i.e. one potential user on top of all existing travellers, who have already made their choices. Still, the equilibrium between the marginal traveller and the operator determines the toll, and this toll determines the number of road users, based on the demand curve. This means that although only one traveller takes an active part in the game, the resulting equilibrium also determines the choices of all other travellers.

The games presented by Bjørnskau and Elvik (18) and Albert (19) are similar in the sense that the traveller and the authority make simultaneous decisions. In the model brought by Bjørnskau and Elvik simultaneity makes sense because the enforcer can decide at any moment whether or not to send enforcers, so the travellers cannot know in advance whether enforcement takes place. In Albert’s game, an explanation of why the toll road authority and the driver act simultaneously is absent. The government in Albert’s game is in fact a Stackelberg leader that acts before the simultaneous choice of the other players, but this issue is not addressed and there is no definition of the government’s objective.

These two games resemble each other also because they both confront an authority with a single traveller. Nevertheless, there is a principal difference between them in terms of the definition of this traveller. Bjørnskau and Elvik assume that the choice is made by all drivers equally and independently. Since the solution is in mixed strategies, the driver’s equilibrium probabilities of choosing each strategy also determine the distribution of choices across all drivers. Albert makes an opposite assumption: the number of travellers that have already chosen the toll road directly affects the utilities of all parties, and the marginal user that participates in the game does not represent anybody else. The input demand curve defines the relations between the toll and the number of road users; hence through the effect of the marginal user on the equilibrium toll, he/she affects the number of toll road users.

Most of the games between travellers and authorities are different from these two games in two manners. First, these are Stackelberg games, where the choice is not simultaneous. The authorities, which are first to choose, base their decision on the predicted reaction of the travellers. Second, the player against the authorities is a composite body that consists of all the travellers.

The first salient Stackelberg game between the collective of travellers and an authority is brought by Fisk (8). Fisk presents a signal control optimization model, in which an authority desires to minimize the total networkwide travel time by setting the control programs in all intersections, and each traveller wants to minimize his/her own travel time by choosing a path through the road network. The problem is formulated as a bi-level program:

\[
\min_{s,f} P(s, f) = \sum_l f_l c_l(f, s)
\]

s.t.

\[
z(f, s) = \min \sum f_l \int_0^l c_l(x, s)dx
\]

where \(s\) is a vector of control variables; \(f\) is a vector of traffic volumes; \(l\) is an index for the network links; and \(c\) is the link travel time or cost. Note that the authority’s objective function is in the upper level and the travellers’ problem, which is actually an assignment problem, is in the lower level. The authority predicts travellers’ reaction to each feasible set of control variables, and chooses the set that will make the travellers choose a pattern of paths that is optimal from the authority’s viewpoint. The importance of Fisk’s game is that it combines, within a NCGT framework, a system-optimum problem and a user-equilibrium problem. It enables the authority to seek the best possible traveller behaviour pattern out of all patterns that meet user equilibrium conditions.
Chen and Ben Akiva (20) develop another model that combines a control problem and assignment; instead of the static model that Fisk used, they present a dynamic model. A dynamic control optimisation problem and a dynamic user optimum problem are formulated separately, and then combined into a game in three different ways: simultaneous solution i.e. Cournot equilibrium; solution where the users’ problem is a constraint in the control program, i.e. Stackelberg equilibrium; and a solution where the authority determines all variables, i.e. monopoly game. The authors do not explicitly explain in which circumstances the simultaneous equilibrium is expected; it can be interpreted as the outcome when the drivers do not know the signal settings in advance. The Stackelberg equilibrium is the most realistic formulation, since it means that the authority uses an expectation of travellers’ driving routes to examine its control strategy. The monopoly game can mainly be used for comparison with other solutions.

Another game between authorities and the collective of travellers is described by Reyniers (21). The players are a railway service operator and the passengers. The operator has to decide how to divide the space on the trains between two service classes, and to determine the fare in each class; the passengers decide which class they will use. The model assumes that Nash equilibrium exists among all passengers and that Stackelberg equilibrium exists between the operator and the passengers. The game does not have a typical NCGT formulation; but it still is another example for a situation where the upper-level player (the operator) bases its decision on a prediction of the behaviour of the lower-level player (all passengers).

Yang and Woo (22) formulate a game that includes, apart from all the travellers, more than one authority. They develop a bi-level program featuring two toll road operators and the travellers. Each operator wishes to maximize its profits by determining the road capacity and the toll. The competition between the operators is in itself a game with Nash equilibrium, described in the upper level. Traffic flows need to meet user equilibrium conditions, determined in the lower level.

Yang and Zhang (23) present another game that involves all travellers and two authorities. The model attempts to estimate the level of penetration of a device that provides information to car drivers. The travellers choose between three travel modes: a car equipped with the device, an unequipped car, and public transport; Nash equilibrium is assumed to determine the distribution of choices. A public transport operator, and a firm that provides the information to users of equipped cars, are involved in the game and wish to maximise their profits. The operator chooses the service frequency and fare, and the information provider determines the data accuracy and the service charge; Nash equilibrium is assumed also between these firms. This is the only game, among games where authorities play against all travellers, where the choice of authorities and travellers is always assumed to be simultaneous. This is plausible given that the game is introduced as a tool for estimating the level of success of a new technology: the provider of this technology must take feedback from the travellers in order to maximise its profit; it prefers to play the game simultaneously with the travellers, because if it were a Stackelberg game, the provider would not be the only leader.

Van Zuylen and Taale (24) present a game between an authority in charge of urban roads, another authority in charge of a ring road, and all travellers. Both authorities need to determine their signal control settings. The urban road authority wishes to minimise the total time spent on its subnetwork; the ring road authority want to maximise the average speed on the ring road; and the travellers try to minimise their individual travel times. The objective functions of the three players are combined in several alternative ways. One possible formulation is when both authorities lead and the travellers follow. Two alternative structures are when one of the authorities is given the power to choose first, and then both the travellers
and the other authority follow. In another formulation, both authorities unite their objectives into a single system optimum, thus reducing the game into a two-player Stackelberg game. The option of simultaneous choice with Nash equilibrium is examined too, although is not considered realistic.

Lim et al (25) formulate a network design problem as a Stackelberg game where the upper-level player is the network designer and the lower-level player is all travellers. The formulation is very general as the design variables can be control settings, information provision, tolls and more. A unique feature of this formulation is that a stochastic user equilibrium problem is used to determine travellers’ choices.

Hollander et al (26) present a Stackelberg game, formulated as a bi-level program, which aims at determining the amount of parking in an urban area. The Stackelberg leader is the authority that sets the amount of parking; the follower is the collective of all travellers to the urban centre. The authority wishes to encourage public transport ridership by introducing parking restrictions, but it fears that if these restrictions are too strict, an excessive amount of travellers will choose to travel to other areas instead of changing their transport mode; the game is aimed at finding the optimal trade-off between these two trends. The objective of the collective of travellers is to reach a distribution of choices which is similar to the distribution determined by a given choice model; the objective of the authority is to maximise the share of travellers who choose a predetermined set of destinations and modes.

Examination of all games between authorities and the collective of travellers shows that most of them describe situations where the authorities seek some systemwide optimum, while the public imaginarily aspires for user equilibrium. Most of these games are formulated as bi-level programs, and the common assumption (at least in the realistic formulations) is that the authorities make their choice first and the travellers follow. Each possible policy of the authorities leads to a different user equilibrium; from the range of equilibrium solutions, the authorities choose the one that meets their objectives best. The result is a compromise, which constitutes a powerful instrument game. The fact that a recommended policy is a direct output of the game (as opposed to games where the authority is not one of the players) is a key feature.

It is important to understand why the player is so often defined as the collective of travellers, while the actual choices are made by individuals. When the player is defined as a single traveller, some explanation is necessary regarding the way that all the other travellers behave. Each one of the following is a possible explanation:

a. All travellers behave equally; they choose the strategy that gives maximum utility. Although such deterministic assumption is often used in NCGT literature, it is too simplistic for most transport applications.

b. The solution is in mixed strategies, and the probability for choosing each strategy also stands for the percentage of travellers who choose it. Such approach is used by Bjørnskau and Elvik (18). This seems to match the probabilistic nature of travellers’ choices, but is still generally unsuitable for transport analysis. One reason for this is that in mixed-strategy equilibrium there is usually a linear relationship between the utility of each strategy and the percentage of travellers who choose it, while in transport choice models a more complex relationship is normally assumed. Another reason is that the technique of finding mixed-strategy equilibrium is based on the assumption of simultaneous choice; but the most plausible assumption here is that the authorities act first.

c. The player is a specific traveller (such as a marginal traveller in Albert (19)), and the behaviour of all other travellers is explained by an external model. But in most cases, such model is not available.
Therefore, when the distribution of traveller choices is important, defining the player as a single traveller brings in serious difficulties, that can be tackled by defining the player as the collective of travellers. Accounting for complex choice distributions is easier through such formulations, because the strategies are defined differently: each strategy of the collective player is a feasible distribution of choices across all travellers. This enables considering complex distributions because each seemingly-deterministic choice of one strategy as the solution of the game is actually a probabilistic solution that involves a diversity of choices. Defining all travellers as a single player can thus solve a major problem, but another problem arises from the fact that this player is an imaginary entity. Imaginary players per se are not uncommon in NCGT models, as we illustrated earlier. The difficulty when the single player signifies all travellers stems from the absence of an explicit will that can be ascribed to this player. In the real world, each traveller has his/her own objective, typically to minimise the total journey time or generalised cost. As we have seen, it is sometime possible to devise a mathematical expression that aggregates the objectives of multiple travellers, and to use this expression as the objective function of the collective player. This objective function should be one that reaches optimum when the decision variables of the collective player describe the most likely results of the interaction between the real travellers, which are not directly involved in the game. But obviously, developing such expression is not easy.

A separate issue that deserves attention is the fact that most games between authorities and the collective of travellers lie on the border between NCGT and other mathematical theories. All the bi-level, Stackelberg games presented here are in fact mathematical programs with equilibrium constraints (MPECs), which is a form familiar to mathematicians; any Stackelberg game is, by definition, an optimisation problem of the leading player. The relations between the group of mathematical models called MPECs and the group called non-cooperative games deserves a separate discussion; none of the two groups contains the other, although they share the same solution techniques. However, note that stating that any of the models reviewed above is a MPEC leads to an analysis with a different focus from when it is stated as a game. If a model is treated as a decision-support instrument, seeing it as MPEC is simpler: it does not require adopting the abstract definitions of the collective player and its aggregate strategies, which in practical-minded circumstances might be considered an intellectual exercise. On the other hand, recognising that the same model is also a game establishes the theoretical foundations for the discussion about equilibrium and enriches the scope for future development.

CONCLUSION

We presented review and analysis of four groups of NCGT models that appear in the transport literature. Table 2 summarises characteristic of all the reviewed games. Table 3 summarises features of the four groups of games. One of the main reasons why modellers are appealed to develop NCGT formulations is the elegance of NCGT models. The identification and formulation of real-life problems as games of an aesthetic structure can lead to valuable insights. A conclusion from our analysis is that elegance mainly characterises the models that we tagged as concept games. The complete conflict of interest between players in games against a demon, the fine symmetry in games between travellers, and the compactness of games between authorities and a single traveller – these are very helpful in pointing to various principles, although they do not develop into applicable methods as they only discuss simple, small-scale examples.

In contrast, games where authorities play against a collective of all travellers, normally formulated as bi-level programs, do not have features that please the eye, but they exhibit
greater applicability. Although these games are more concerned about providing practical results than about enriching the theory, their recognition as games has a conceptual importance in itself; from a strict application-oriented perspective, they can be equally treated as mathematical programs with equilibrium constraints.

Some capabilities of NCGT, which are of great importance in transport analysis, are hardly used by any of the reviewed games. These games assume that all players have full knowledge of their own strategies and payoffs, as well as knowledge of the strategies and payoffs of the other players. Games with incomplete knowledge, either borrowed from other fields or purposely-formulated to describe transport problems, can yield more realistic insights when modelling behaviour under uncertainty. Another issue, that the presented classifications do not cover, is the distinction between games describing observable system states and games where some strategies constitute circumstances that are still unfamiliar. Only few games deal with innovative policies (13 and 19, if we see tolling as innovative) or technological advances (23). One might suggest that modellers do not see the equilibrium concept of NCGT as established enough to handle the risky analysis of futuristic scenarios.

Still, as our review discloses, NCGT models have potential for further development of reliability estimators; for modelling trends such as competition and tendering; and for creation of systematic policy-oriented design procedures, that aim at optimising networkwide performance while accounting for the expected travellers’ response.

ACKNOWLEDGMENT

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## Table 2: Characteristics of the reviewed games

(Continues on next page)
<table>
<thead>
<tr>
<th>Source</th>
<th>Players</th>
<th>Strategies / decision variables</th>
<th>Objectives</th>
<th>Format</th>
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<tbody>
<tr>
<td>Fisk (8)</td>
<td>Authority</td>
<td>Traffic control settings</td>
<td>Minimise total network time / fuel consumption</td>
<td>Stackelberg program</td>
</tr>
<tr>
<td></td>
<td>Drivers</td>
<td>Route choice</td>
<td>Minimise time</td>
<td></td>
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<tr>
<td>Chen and Ben Akiva (20)</td>
<td>Authority</td>
<td>Traffic control settings</td>
<td>Minimise total network time</td>
<td>Cournot / Stackelberg / Monopoly, systems of inequalities</td>
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<td></td>
<td>Drivers</td>
<td>Route choice</td>
<td>Minimise time</td>
<td></td>
</tr>
<tr>
<td>Reyniers (21)</td>
<td>Railway operator</td>
<td>Capacity and fare of 1\textsuperscript{st}/2\textsuperscript{nd} class trains</td>
<td>Maximise profit</td>
<td>Stackelberg, not standard</td>
</tr>
<tr>
<td></td>
<td>Passengers</td>
<td>1\textsuperscript{st} class / 2\textsuperscript{nd} class</td>
<td>Minimise density and cost</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Drivers</td>
<td>Route choice</td>
<td>Minimise cost</td>
<td>Stackelberg program (Nash equilibrium between operators)</td>
</tr>
<tr>
<td>Yand and Woo (22)</td>
<td>Toll road operators a and b</td>
<td>Capacity and toll in road a and b</td>
<td>Maximise profit</td>
<td>Program (Nash equilibrium between travellers and between firms)</td>
</tr>
<tr>
<td></td>
<td>Drivers</td>
<td>Route choice</td>
<td>Minimise cost</td>
<td></td>
</tr>
<tr>
<td>Yand and Zhang (23)</td>
<td>Advanced information provider</td>
<td>Data accuracy and service charge</td>
<td>Maximise profit</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Public transport operator</td>
<td>Service quality and fare</td>
<td>Maximise profit</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Travellers</td>
<td>Equipped car / unequipped car / public transport</td>
<td>Minimise time and cost</td>
<td></td>
</tr>
<tr>
<td>Van Zuylen and Taale (24)</td>
<td>Urban roads authority</td>
<td>Traffic control settings on urban roads</td>
<td>Minimise urban roads time</td>
<td>Several alternative Stackelberg programs (and other programs)</td>
</tr>
<tr>
<td></td>
<td>Travellers</td>
<td>Route choice</td>
<td>Minimise time</td>
<td></td>
</tr>
<tr>
<td>Lim et al (25)</td>
<td>Network designer</td>
<td>Various network design parameters</td>
<td>Minimise total network cost</td>
<td>Stackelberg program</td>
</tr>
<tr>
<td></td>
<td>Travellers</td>
<td>Route choice</td>
<td>Minimise time</td>
<td></td>
</tr>
<tr>
<td>Hollander et al (26)</td>
<td>Authority</td>
<td>Parking policy</td>
<td>Improve urban centre vitality and public transport ridership</td>
<td>Stackelberg program</td>
</tr>
<tr>
<td></td>
<td>Travellers</td>
<td>Mode and destination</td>
<td>Maximise any utility function</td>
<td></td>
</tr>
</tbody>
</table>

Table 2: Characteristics of the reviewed games
(Continued from previous page)
<table>
<thead>
<tr>
<th>Type</th>
<th>Structure</th>
<th>Choice order</th>
<th>Level of competitiveness between players</th>
<th>Principles and assumptions</th>
<th>Contribution</th>
<th>Potential areas for future studies</th>
</tr>
</thead>
<tbody>
<tr>
<td>Games against a demon</td>
<td>Zero sum: the players minimise/ maximise the same objective</td>
<td>Simultaneous</td>
<td>Very high</td>
<td>A demon tries to thwart the traveller’s journey</td>
<td>Estimates of worst-case scenario</td>
<td>Reliability estimation</td>
</tr>
<tr>
<td>Games between travellers</td>
<td>Symmetrical: all players have equal objectives and strategies</td>
<td>Simultaneous     (except one case in Pedersen, 2003)</td>
<td>High</td>
<td>Competition between users on road space: additional users cause reduced level of utility for everyone. Similar to the Prisoner’s Dilemma</td>
<td>Mainly conceptual models of user equilibrium</td>
<td>Various choice models (mode, destination etc.) but only for small-scale demonstration</td>
</tr>
<tr>
<td>Games between authorities</td>
<td>Varies</td>
<td>Either simultaneous or leader-follower</td>
<td>Varies</td>
<td></td>
<td>Very few games found in literature</td>
<td>Modelling competition between firms and hierarchy between authorities</td>
</tr>
<tr>
<td>Games between travellers and authorities</td>
<td>Not elegant: the players’ objectives are different but not the opposite of each other</td>
<td>Mainly leader-follower</td>
<td>Low</td>
<td>When the player is a single traveller: various simple assumptions regarding choice distribution</td>
<td>When the player is a single traveller: conceptual</td>
<td>None</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>When the player is all travellers: realistic choice distributions</td>
<td>When the player is all travellers: practical, compromise between system optimum and user equilibrium</td>
<td>Policy games: design of network layout and transport services to meet designer’s objectives while considering expected travellers’ reaction</td>
</tr>
</tbody>
</table>

Table 3: Groups of games
REFERENCES


